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No cosmological domain wall problem for weakly coupled fields

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Abstract

After inflation occurs, a weakly coupled scalar field will in general not be in thermal equilibrium but have a distribution of values determined by the inflationary Hubble parameter. If such a field subsequently undergoes discrete symmetry breaking, then the different degenerate vacua may not be equally populated so the domain walls which form will be 'biased' and the wall network will subsequently collapse. Thus the cosmological domain wall problem may be solved for sufficiently weakly coupled fields in a post-inflationary universe. We quantify the criteria for determining whether this does happen, using a Higgs-like potential with a spontaneously broken Z_2 symmetry.

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I. INTRODUCTION

It is generally believed that if *discrete* symmetries of scalar fields are spontaneously broken as the universe cools down, then there would be severe difficulties for its subsequent evolution [1]. This is because topological defects — domain walls — would form at the boundaries of the different degenerate vacua chosen in causally disconnected regions following the symmetry breaking phase transition [2] and would eventually come to dominate the total energy density, in conflict with observations [3]. To avoid this requires the energy scale of the symmetry breaking phase transition to be lower than ~ 100 MeV; in fact it must be less than ~ 1 MeV if the anisotropy induced by the walls in the cosmic microwave background radiation is to be below experimental limits [1,3]. This is a severe constraint on attempts to extend physics beyond the Standard Model, which often involve introducing such discrete symmetries [4].

In some special circumstances, the broken discrete symmetry may not be restored at high temperature so domain walls would never form [5]. There would also appear to be no problem if the symmetry breaking occurs prior to inflation since one would then expect the density of any resulting topological defects to be exponentially diluted away. However defects can still form in this case through quantum fluctuations of the scalar field *during* inflation [7–9] if the mass of the field is less than the inflationary Hubble parameter. When inflation is driven by a F-term supergravity potential, the breaking of supersymmetry by the large vacuum energy gives all scalar fields, including the inflaton itself, a mass-squared of $\mathcal{O}(H^2)$ [13,14]; in this case fluctuations are negligible and walls will *not* form. However if we consider instead e.g. a D-term or no-scale inflationary potential [6], the scalar field may remain light relative to the Hubble parameter so the above mechanism will be operative and domain walls will form.

Although the first such paper [7] considered an axion field, subsequent work [8,9] has been mainly concerned with scalar fields which have sufficiently strong couplings that the vacuum expectation value (vev) during inflation does not increase much above the Hubble parameter. The field then remains uncorrelated on spatial scales larger than the Hubble radius and defects form during or at the end of inflation. However very weakly coupled scalar fields are arguably of more interest in cosmology. For example the field responsible for driving inflation should have very small couplings in order that its quantum fluctuations not contribute excessively to the anisotropy of the cosmic microwave background [6]. There has been much interest in 'quintessence' [10] — a very weakly coupled evolving field that may account for the tiny vacuum energy that is suggested by some astronomical observations. Weakly coupled fields can also be a source of dark matter through their coherent oscillations [11]. In a recent paper [12] an extremely weakly coupled dilaton field that forms domain walls is proposed as a way of binding the matter in spiral galaxies and producing their characteristic flat rotation curves (as an alternative to cold dark matter). Particularly in the context of this model, it is interesting to ask whether the above mechanism would indeed create stable domain walls.

The point is that such a very weakly coupled field will be *correlated* on super-horizon scales at the end of inflation and not be brought back into thermal equilibrium during the reheating process since it has no couplings to the thermal plasma or to the inflaton. The field will oscillate coherently during the post-inflationary Friedman-Lemaitre-Robertson-Walker (FLRW) expansion era and when the expansion redshift reduces the energy in its coherent oscillations it will settle into different symmetry-breaking vacua on spatial scales larger than the Hubble radius, thus forming defects. It would be likely for the same vacuum to be chosen in different (apparently causally

disconnected) regions. A 'bias' could thus be generated in the probabilities for populating the distinct vacuua even if they are energetically degenerate [15]. After the walls form, such a bias, even if very small, will result in exponential decay of the wall network, as has been demonstrated both analytically and numerically [16–18]. Thus there may be no domain wall problem for weakly coupled fields in a post-inflationary universe.

Our aim is to quantify the bias that would be created for such a hypothetical field with specified properties in order to determine the fate of the domain walls formed. We consider the problem in its simplest form and focus on domain wall formation through inflationary fluctuations in a spontaneously broken Z_2 theory of a real scalar field with the Higgs-like potential

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2\,, \tag{1}$$

where $v = m/\sqrt{\lambda}$. First we study (Section II) the stochastic evolution of the field perturbations during the inflationary epoch which set the relevant initial conditions. In Section III we follow the evolution of the fluctuations during the FLRW expansion until the field drops into its potential minima and the domain walls are formed. The bias between the degenerate vacua is calculated in Section IV. We review the history of the field evolution in Section V and identify the regions in the parameter space of the above model where domain walls do not survive. Finally we present our conclusions and comment on specific models concerning domain walls such as Ref. [12].

II. STOCHASTIC APPROACH FOR THE INITIAL CONDITIONS

During inflation the smooth component of a slowly evolving scalar field can be considered (on scales larger than the horizon) to be a classical variable subject to stochastic noise (contributed by the field modes whose exponentially increasing wavelength causes them to 'exit the horizon', becoming part of the coarse-grained field) [19–22,24]. The Langevin equation governing the coarse-grained field ϕ is [19]

$$\dot{\phi} = -\frac{V'(\phi)}{3H_\mathrm{i}} + \frac{H_\mathrm{i}^{3/2}}{2\pi}\eta(t)\,, \tag{2}$$

where the white noise η satisfies

$$\langle\eta(t)\eta(t')\rangle = \delta(t-t')\,. \tag{3}$$

This equation can be restated as a Fokker-Plank equation for the probability distribution $P(\phi,t)$ of the field values in a given coarse-grained domain [19]:

$$\frac{\partial P(\phi,t)}{\partial t} = \frac{\partial}{\partial\phi}\left(\frac{1}{3H_\mathrm{i}}\frac{\partial V}{\partial\phi}P(\phi,t)\right) + \frac{H_\mathrm{i}^3}{8\pi^2}\frac{\partial^2 P(\phi,t)}{\partial\phi^2}\,. \tag{4}$$

Here H_i, the Hubble parameter during inflation, is taken to be independent of ϕ, i.e this field is assumed *not* to contribute significantly to the vacuum energy during inflation. (In the analogous equation for the inflaton field, H_i is itself a function of ϕ giving rise to ordering ambiguities in the corresponding Fokker-Planck equation.)

If the field evolution is 'slow-roll', then the force term (involving the potential) can be neglected compared with the noise term in Eq.(4). Then starting from a given value of the field $\phi = \bar{\phi}$ averaged

over a patch of size H_i^{-1} when cosmologically relevant scales 'exit the horizon', the solution to this equation is the Gaussian distribution:

$$P_\phi \equiv P(\phi, \bar{\phi}, t) = \sqrt{\frac{8\pi^3}{H_\mathrm{i}^3 t}} \exp\left[-\frac{2\pi^2 \left(\phi - \bar{\phi}\right)^2}{H_\mathrm{i}^3 t}\right], \tag{5}$$

where H_i is taken to be approximately constant as is required for successful inflation [6].

The mean value $\langle\phi^2\rangle = H_\mathrm{i}^3 t/4\pi^2$ grows linearly with time as in Brownian motion [25]. Relating the time during inflation to the cosmological scale through $l^{-1} \sim H_\mathrm{i}\mathrm{e}^{-H_\mathrm{i}t}$, we can write the probability distribution as [7,15]

$$P_\phi(\phi, \bar{\phi}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2\sigma^2}\left(\phi - \bar{\phi}\right)^2\right], \tag{6}$$

where σ^2 is the quadratic dispersion of the field. This can be obtained, with reference to the noise term in Eq.(2), as the sum of independent Gaussian distributions with dispersion-squared $(H_\mathrm{i}/2\pi)^2$, one for each e-fold of inflation. The sum is to be taken over the period when scales between l_min and l_max leave the horizon, where l_min corresponds to the Hubble radius at some moment during the FLRW era (representing an ultraviolet cutoff, given that we are interested in the super-horizon behavior) and l_max is the biggest spatial scale of interest, i.e. of order the present Hubble radius H_0^{-1}. The total dispersion-squared is then just the sum of the dispersion-squared for the independent probabilities:

$$\sigma^2 = \frac{H_\mathrm{i}^2}{4\pi^2}\int_{l_\mathrm{min}}^{l_\mathrm{max}} \mathrm{d}\log l = \frac{H_\mathrm{i}^2}{4\pi^2}\log\left(\frac{H_0^{-1}}{l_\mathrm{min}}\right), \tag{7}$$

i.e. of $\mathcal{O}(H_\mathrm{i}^2)$ in the cases of interest.

The formula (6) assumes that the value of the force term in Eq.(2) is negligible in comparison with the noise term so the value of $\bar{\phi}$ is not determined. However, if inflation continues for a large number of e-folds the force term will impede the tendency of the distribution to widen indefinitely. In this case stochastic equilibrium is achieved and it is possible to give a probabilistic prediction for the initial $\bar{\phi}$ using the *stationary* solution for the Fokker-Planck equation (while Eq.(6) still gives the distribution for the field at the end of inflation on cosmologically interesting scales).[1]

The stationary case $\partial P/\partial t = 0$ can be solved to obtain the probability distribution of the averaged field $\bar{\phi}$:

$$P_{\bar{\phi}} = C_1 \exp\left(-\frac{8\pi^2}{3}\frac{V(\bar{\phi})}{H_\mathrm{i}^4}\right) + C_2 \exp\left(-\frac{8\pi^2}{3}\frac{V(\bar{\phi})}{H_\mathrm{i}^4}\right)\int_0^{\bar{\phi}} \exp\left(\frac{8\pi^2}{3}\frac{V(\phi')}{H_\mathrm{i}^4}\right)\mathrm{d}\phi'. \tag{8}$$

If the potential is an even function the first term is also even while the second term is odd. Therefore, if the potential remains positive for large ϕ, the second term will have greater absolute values than the first at some point, and as it is an odd function the probability would have negative values. This shows that the second term is unphysical. Thus, in the stationary case the normalized probability distribution is just:

[1]This has been shown in studies of 'eternal inflation' [26].

$$P_{\bar\phi} = \exp\left(-\frac{8\pi^2}{3}\frac{V(\bar\phi)}{H_\mathrm{i}^4}\right) \Bigg/ \int_{-\infty}^{+\infty} \exp\left(-\frac{8\pi^2}{3}\frac{V(\phi')}{H_\mathrm{i}^4}\right) \mathrm{d}\phi' \ . \tag{9}$$

Defining the dimensionless normalized fields $\chi \equiv \phi/v$, and $\bar\chi \equiv \bar\phi/v$, this is a one-parameter function for our chosen potential (1):

$$P_{\bar\chi} = \frac{2}{\pi\left[I(\frac{1}{4},\frac{\pi^2}{3\beta^4}) + I(-\frac{1}{4},\frac{\pi^2}{3\beta^4})\right]} \exp\left[-\frac{2\pi^2}{3\beta^4}\left(\bar\chi^4 - 2\bar\chi^2 + \frac{1}{2}\right)\right], \quad \beta \equiv \lambda^{-1/4}\frac{H_\mathrm{i}}{v}\,, \tag{10}$$

where $I(x,y)$ is the modified Bessel function of first kind. This is plotted in Fig.1 for various values of β.

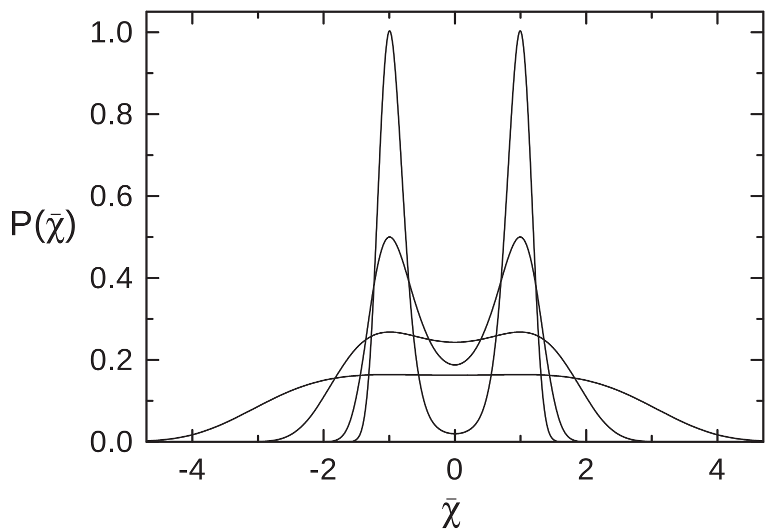

FIG. 1. Stochastic equilibrium probability distribution for the (normalized) field vev during inflation. The curves shown are for $\beta = 1.2, 1.6, 2.8, 5$ in order of decreasing height.

Since the height of the potential barrier at the origin separating the two vacua is $h^4 = \frac{1}{4}\lambda\, v^4$ from Eq.(1), the parameter β in Eq.(10) is just $\beta = H_\mathrm{i}/\sqrt{2}h$. Thus for small β the probability of transitions between the potential wells is exponentially suppressed; the discrete symmetry is broken during inflation and the field is localized in one minimum or the other so domain walls will not form (assuming that the symmetry is not restored during reheating after inflation). However when $H_\mathrm{i} \gg h$ the fluctuations are large enough to make the probability distribution flat as seen in Fig.1.

In general the average value of the potential energy in the field ϕ is

$$\langle V\rangle = \int V(\bar\phi) P_{\bar\phi}\, \mathrm{d}\bar\phi = g(\beta) H_\mathrm{i}^4\,, \tag{11}$$

where $g(\beta)$ is a smooth positive function with maximum value 0.02 and limiting values 1.9×10^{-2} for $\beta \to 0$ and 9.4×10^{-3} for $\beta \to \infty$. Thus the energy density of the scalar field under consideration is indeed negligible when compared with that of the inflaton $V_\mathrm{i} \simeq 3H_\mathrm{i}^2 M_\mathrm{P}^2$, as was implicitly assumed in neglecting the ϕ dependence of H_i. (Here $M_\mathrm{P} \equiv 1/\sqrt{8\pi\, G} \simeq 2.4\times10^{18}$ GeV is the normalized Planck scale.) Note that the COBE observations of large-scale anisotropy in the cosmic microwave background set a strict upper bound on the Hubble parameter during inflation [6]

$$V_{\mathrm{i}}^{1/4} \ll 0.027 M_{\mathrm{P}} \Longrightarrow \frac{H_{\mathrm{i}}}{M_{\mathrm{P}}} \ll 4.2 \times 10^{-4}. \tag{12}$$

This still allows the energy scale of inflation to be as high as the GUT scale ($\sim 10^{16}$ GeV) but in specific models it can be much lower than this, in particular in 'new' inflationary models with a quadratic leading term in the potential [28].

The average value of $|\bar{\phi}|$ is just v for small β, while for large β it is $\sim 0.305\, H_{\mathrm{i}}/\lambda^{1/4}$. Thus if $\lambda \lesssim 8 \times 10^{-3}$, the field vev grows above H_{i}. The number of e-folds of inflation that are necessary to achieve the stationary distribution in this case must exceed $(\langle\bar{\phi}\rangle/\delta\phi)^2 \simeq \lambda^{-1/2}$, taking the field increment per e-fold to be $\delta\phi \simeq H_{\mathrm{i}}/2\pi$. Otherwise one cannot predict an unique probability distribution for the field.

The slow-roll condition $\ddot{\phi} \ll 3H_{\mathrm{i}}\dot{\phi}$ implies $|V''(\phi)| \ll 9H_{\mathrm{i}}^2$. For the potential (1) and the distribution (10) this translates into $m^2 \ll \frac{9}{2}H_{\mathrm{i}}^2$ if the quadratic term in the potential dominates (small β). When the quartic term dominates (large β) one must require $\lambda \lesssim 1$. If the slow-roll conditions are not obeyed, the classical stochastic treatment does not apply and the quantum creation of particles is exponentially suppressed [23]. For the case under consideration, $h \ll H_{\mathrm{i}}$, the relevant slow-roll condition is $\lambda \lesssim 1$ and these two conditions together imply $m \ll H_{\mathrm{i}}$.

III. THE EVOLUTION DURING THE FLRW ERA

In Ref. [15] it was shown that the probability distribution for the coarse-grained field outside the horizon during the FLRW era following the inflationary era evolves into a non-Gaussian distribution. We are interested here in obtaining a discrete probability distribution for the two vacua of the field. We follow the evolution of the field from its initial value ϕ_o at the end of inflation until it settles down into one of its discrete minima. By obtaining the distribution function $f(\phi_o)$ whose values are ± 1 according to the vacuum finally chosen, we can compute the bias between the vacua.

The equation of motion for the modes of a scalar field outside the horizon during the FLRW era is

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} + 3\frac{c}{t}\frac{\mathrm{d}\phi}{\mathrm{d}t} + V' = 0\,, \quad \text{where} \quad c = Ht\,. \tag{13}$$

We have rewritten the Hubble parameter in terms of the variable c which equals $1/2\,(2/3)$ for a radiation- (matter-) dominated universe. The initial conditions are $c/t_0 = H_{\mathrm{i}}$, $\phi = \phi_o$, and $\dot{\phi}_0 = 0$. In general there is no exact solution for this equation, however a good approximation may easily be obtained [27].

Let us first suppose that the friction term is relatively unimportant so at first approximation the field oscillates in its potential minimum at the end of inflation according to the Lagrangian $L = \frac{1}{2}\dot{\phi}^2 - V$. Then energy conservation implies

$$\dot{\phi}^2 = 2(V_{\mathrm{max}} - V)\,, \tag{14}$$

where V_{max} is the maximum value of the potential energy during the oscillation. Thus the oscillation period is given (for a symmetric potential) by

$$\Delta t = 4\int_0^{\phi_{\mathrm{max}}} \frac{\mathrm{d}\phi}{\sqrt{2(V_{\mathrm{max}} - V)}}\,. \tag{15}$$

When $H \ll \omega = 2\pi/\Delta t$ the friction term in Eq.(13) is ω/H times smaller than the other terms so the assumption of negligible friction is valid. Until H drops below ω the field remains approximately fixed since the friction is relatively high and the dynamical time exceeds the expansion age H^{-1}. After this point the field starts oscillating, losing energy density of order

$$\Delta\rho = \oint 3H\dot{\phi}\,\mathrm{d}\phi = 12H\int_0^{\phi_{\max}}\sqrt{2(V_{\max}-V)}\mathrm{d}\phi \tag{16}$$

in each oscillation. Here we have used the fact that in this regime H is approximately constant during each oscillation period.

For future use we also consider the case of a general power law potential

$$V(\phi) = \frac{\lambda}{\gamma}\phi^{\gamma}\,. \tag{17}$$

Let ϕ_n be the values of ϕ that are turning points of the trajectory, t_n the corresponding times and ρ_n the corresponding energy densities. Evidently $\phi_n \to 0$ and t_n grows, eventually going to infinity. Equations (15) and (16) imply the following relations between these quantities

$$\Delta\phi_n = \frac{\Delta\rho_n}{\lambda\phi_n^{\gamma-1}} = -\frac{k_2}{t_n}\phi_n^{2-\gamma/2}\,, \tag{18}$$

$$\Delta t_n = k_1\phi_n^{1-\gamma/2}\,, \tag{19}$$

where $k_1 = \left(\frac{2\sqrt{2\pi\gamma}\,\Gamma(1+1/\gamma)}{\Gamma(1/2+1/\gamma)}\right)\lambda^{-1/2}$ and $k_2 = c\left(\frac{12\sqrt{2\pi}\,\Gamma(1/\gamma)}{\sqrt{\gamma}(2+\gamma)\Gamma(1/2+1/\gamma)}\right)\lambda^{-1/2}$. The solution to these recurrence relations are the power-laws:

$$\phi_n = \phi_1 n^a, \quad a = -\frac{6c}{2(1+3c)+\gamma(1-3c)}\,, \tag{20}$$

$$t_n = t_1 n^b, \quad b = \frac{(2+\gamma)}{2(1+3c)+\gamma(1-3c)}\,.$$

The energy density is $\rho_\phi \sim \phi^\gamma \sim t^{a\gamma/b}$. In terms of the cosmological scale-factor R this can be written, $\rho_\phi \sim R^{a\gamma/bc}$, i.e.

$$\rho_\phi \sim R^{-6\gamma/(2+\gamma)}\,, \tag{21}$$

independently of c [27]. The number of oscillations goes as $n \sim \phi^{1/a} \sim t^{1/b} \sim R^{1/cb}$. Table I shows the exponents of different quantities as functions of n, R and t for the cases of radiation- and matter- domination and for quadratic and quartic potentials.

Let us return to the double well potential (1) which interests us here. As we saw in the preceding Section, the quartic term dominates at the end of inflation in the cases of interest, i.e $\beta \gg 1$. The subsequent evolution starting with $\phi = \phi_0$ at $t = t_0$ goes as follows. The field oscillates in the quartic term-dominated potential (where the mass term can be neglected), decreasing in amplitude due to friction caused by the Hubble expansion, until $V^{1/4}$ drops below the height of the barrier h separating the two vacua, or in other words ϕ becomes of $\mathcal{O}(v)$, and the field settles down in one of its potential minima. The energy density in oscillations then decreases as for radiation ($\rho_\phi \sim R^{-4}$) independently of the rate of expansion. The relation between the number of half-oscillations and the field amplitude is $n \sim \phi^{-1}$ if the universe is radiation-dominated and $n \sim \phi^{-1/2}$ if it is matter-dominated. Thus ϕ will be found around one of its vevs $\pm v$ after having completed $n = \tau_{\mathrm{r}}\chi_0$

half-oscillations in the radiation-dominated case and $n = \tau_{\rm m}\sqrt{\chi_0}$ in the matter-dominated case, where $\chi_0 = \phi_0/v$, and $\tau_{\rm r}$, $\tau_{\rm m}$ are constants. (The numerical values of these constants cannot be calculated given the approximations we have made since they depend on the details of the evolution towards the end of the oscillations (when the mass term begins to be significant), and even more importantly on $t_0\,(= c/H_{\rm i})$, which sets the Hubble parameter at the beginning of the oscillations and thus determines the friction term.) In general n is a function of χ_0, t_0 and λ. However, rewriting the equation (13) using $\hat\phi = \phi/\phi_0$ and $\hat t = \sqrt{\lambda}\phi_0\, t$, we have:

$$\frac{{\rm d}^2\hat\phi}{{\rm d}\hat t^2} + 3\frac{c}{\hat t}\frac{{\rm d}\hat\phi}{{\rm d}\hat t} + \left(\hat\phi^3 - \frac{1}{\chi_0^2}\hat\phi\right) = 0\,. \tag{22}$$

This equation has one parameter and the initial conditions are $\hat\phi_0 = 1$, ${\rm d}\hat\phi/{\rm d}\hat t = 0$. Therefore the number of oscillations must depend on just χ_0 and on $\hat t_0 = c\sqrt{\lambda}\phi_0/H_{\rm i}$. However, as we stated before, if initially $H_{\rm i} \gg \omega = 0.84\sqrt{\lambda}\phi_0$ the field remains frozen. It only begins to oscillate when H decreases below ω, i.e. at $\hat t_0 \sim 1$, hence the problem does not depend on $\hat t_0$ or any other parameter as long as we are interested only in the final state. We have checked numerically that n does not depend significantly on $\hat t_0$ when $\hat t_0 \lesssim 0.3$. In the present case we have that the initial distribution of ϕ_0 is most probably concentrated around $\phi_0 \sim 0.1\,\lambda^{-1/4}H_{\rm i}$, and $\hat t_0 \lesssim 0.1c\,\lambda^{-1/4}$, so there can be no significant dependence on $\hat t_0$. (It may be that the initial value of the Hubble parameter for the FLRW era, H_0, is somewhat less than its inflationary value, $H_{\rm i}$, but even in this case the results apply for small λ.) The proportionality constants can be calculated numerically in this regime and are found to be $\tau_{\rm r} = 0.31$ and $\tau_{\rm m} = 0.63$. When $\hat t_0$ exceeds unity the Hubble parameter is smaller than ω at the beginning of the oscillations and the reduction of the friction implies a proportional increase of the number of oscillations: $\tau_{\rm r}, \tau_{\rm m} \propto \hat t_0$ for $\hat t_0 > 1$.

The function $f(\chi_0)$ that equals ± 1 according to the value $\phi = \pm\, v$ of the final state will then change sign just once as n increases by unity. Therefore we can write

$$\begin{aligned} f(\chi_0) &= (-1)^{{\rm int}(\tau_{\rm r}\chi_0)}, \quad \text{for} \quad c = \frac{1}{2}\,, \\ &= (-1)^{{\rm int}(\tau_{\rm m}\sqrt{\chi_0})}, \quad \text{for} \quad c = \frac{2}{3}\,, \end{aligned} \tag{23}$$

where ${\rm int}(x)$ is the integer part of x. Small deviations from these expressions occur for low values of χ_0. The oscillations start when $t = t_* \simeq 1/\sqrt{\lambda}\phi_0 \simeq \lambda^{-1/4}H_{\rm i}^{-1}$, and whether the universe is radiation- or matter- dominated at this time is determined by whether t_* is smaller or larger than the epoch of matter and radiation equality $t_{\rm eq}$. (There is also the possibility of a different expansion rate during the reheating process.)

	$\phi(n)$	$t(n)$	$\rho_\phi(n)$	$\phi(R)$	$t(R)$	$\rho_\phi(R)$	$n(t)$	$n(R)$
$c = 1/2$; $\gamma = 2$	-3/4	1	-3/2	-3/2	2	-3	1	2
$c = 1/2$; $\gamma = 4$	-1	2	-4	-1	2	-4	1/2	1
$c = 2/3$; $\gamma = 2$	-1	1	-2	-3/2	3/2	-3	1	3/2
$c = 2/3$; $\gamma = 4$	-2	3	-8	-1	3/2	-4	1/3	1/2

TABLE I. Power-law exponents for the evolution of field variables during oscillations after inflation, for a quadratic ($\gamma = 2$) and quartic ($\gamma = 4$) potential, assuming a radiation-dominated ($c = 1/2$) and matter-dominated ($c = 2/3$) universe.

Note that the functions (23) apply only when the oscillations begin and end during a period of expansion while c is constant. As an example of a more complex situation consider the case where the oscillations start in the radiation-dominated era and end in the matter-dominated one. According to Table I the amplitude of the oscillations goes as $\phi \sim 1/R$. The amplitude at the time of matter-radiation equality is then $\phi_{\rm eq} = \phi_0 (t_*/t_{\rm eq})^{1/2} \sim (\phi_0/t_{\rm eq}\sqrt{\lambda})^{1/2}$, while the condition that the oscillations end in the matter-dominated period is $\phi_{\rm eq} > v$. The number of oscillations in the radiation-dominated period is just $n_{\rm r} \simeq \phi_0/\phi_{\rm eq} = (t_{\rm eq}\sqrt{\lambda}\phi_0)^{1/2}$. Therefore the number of half-oscillations completed in the radiation-dominated period is proportional to $\sqrt{\phi_0}$. With respect to the remaining oscillations that occur in the matter-dominated period, since they start at $t_{\rm eq}$ well inside the low friction regime, the constant $\tau_{\rm m}$ in Eq.(23) has to be scaled by $\hat{t}_{\rm eq} = \sqrt{\lambda}\phi_{\rm eq} t_{\rm eq}$. Accordingly, the matter-dominated period gives the number of oscillations $n_{\rm m} \simeq \sqrt{\lambda}\phi_{\rm eq} t_{\rm eq}\sqrt{\chi_{\rm eq}} = \lambda^{1/8} t_{\rm eq}^{1/4} \phi_0^{3/4}/\sqrt{v}$. Thus, for such mixed situations, different functional dependencies of ϕ_0 are expected in the formula for the number of oscillations.

IV. BIAS

The bias, defined as the difference in the probabilities of populating the two discrete vacua, is given by the convolution

$$b(\bar{\chi}) = \int f(\chi_0)\, P_\chi(\chi_0, \bar{\chi})\, \mathrm{d}\chi_0 \,. \tag{24}$$

The probability distribution of the field values at the end of inflation $P_\chi(\chi_0, \bar{\chi})$ is given by Eq.(6) (rewritten in terms of the normalized fields $\chi = \phi/v$ and $\bar{\chi} = \bar{\phi}/v$), while the function $f(\chi_0)$ that gives the sign of the field in the final vacuum state was obtained in the previous Section for different cosmological situations. (Note that $P_\chi(\chi_0, \bar{\chi})$ is almost independent of spatial scale in the FLRW era since observable scales correspond to only a few e-foldings during inflation.) In the following we do a detailed analysis of the bias function for the cases of radiation- and matter-dominated universes and then present an approximation suitable for more complicated situations.

The bias in the observable universe is well defined by Eq.(24) but since only the probabilistic distribution (10) is available for $\bar{\chi}$, the predictions are also in terms of a probability distribution for the bias which satisfies

$$P_b = \sum P_{\bar{\chi}} \frac{\mathrm{d}\bar{\chi}}{\mathrm{d}b} \,, \tag{25}$$

where $\bar{\chi}$ is understood as a function of b (inverse of the function (24)), and the sum is over the different branches in the solution of the equation $b = b(\bar{\chi})$. The cumulative probability for the bias to be less than some particular value is then just the integral:

$$P(|b| < x) = \sum \int_{b(\bar{\chi})=-x}^{b(\bar{\chi})=x} P_{\bar{\chi}}\, \mathrm{d}\bar{\chi} \,. \tag{26}$$

The problem has basically two parameters,

$$\alpha = \frac{\sigma}{v} \sim \frac{H_{\rm i}}{v} \,, \tag{27}$$

which gives the width of the Gaussian probability distribution (6) in terms of the normalized field $\chi_0 \equiv \phi_0/v$, and $\beta \sim \lambda^{-1/4}\alpha$ (using Eq.(10)) which gives the width of the distribution (10) of the

normalized average field $\bar{\chi} \equiv \bar{\phi}/v$. The conditions we are assuming for slow-roll $m \lesssim H_{\rm i}$ and $\lambda < 1$ imply that $\alpha < \min(\beta, \beta^2)$ but they do not constrain this parameter to be greater or smaller than unity since we also require $\beta \gg 1$ (i.e. $H_{\rm i} \gg h$) in order for the field to be able to jump the potential barrier during inflation.

1. Radiation dominated universe

In this case the number of half-oscillations $n \sim \tau_{\rm r}\,\chi_0$, and $f(\chi_0) = (-1)^{{\rm int}(\tau_{\rm r}\,\chi_0)}$ is a square-wave. It is easy to see that the bias is then a periodic function of $\bar{\chi}$ with period $2\tau_{\rm r}^{-1}$, and can be expanded in the Fourier series

$$b(\bar{\chi}) = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin[(2n+1)\pi\tau_{\rm r}\bar{\chi}]}{(2n+1)}\exp\left[-(2n+1)\frac{\pi\tau_{\rm r}}{\sqrt{2}}\alpha\right]^2\,, \tag{28}$$

which converges exponentially fast. For $\alpha \gtrsim 1$ (i.e. $H_{\rm i} \gtrsim v$) we can approximate the series by its first term

$$b(\bar{\chi}) = \frac{4}{\pi}\sin(\pi\tau_{\rm r}\bar{\chi})\exp\left(-\frac{\pi\tau_{\rm r}}{\sqrt{2}}\alpha\right)^2\,, \tag{29}$$

showing that the bias is a sine function with an exponentially damped amplitude. In the opposite case $\alpha \ll 1$ (i.e. $H_{\rm i} \ll v$), more terms of the series must be added so it converges to the Fourier series for the square wave, i.e. identical to Eq.(28) without the exponential factor. In this limit the Gaussian distribution $P_\chi(\chi_0, \bar{\chi})$ is appropriate inside the regions where $f(\chi_0)$ has one sign or the other, but moving $\bar{\chi}$ from one of these regions to another where $f(\chi_0)$ has *opposite* sign causes the function $b(\bar{\chi})$ to step as the error function, ${\rm erf}(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x {\rm d}t\,{\rm e}^{-x^2}$. Therefore for $\alpha \ll 1$ we can write the periodic function b in the interval $[-\tau_{\rm r}^{-1}, \tau_{\rm r}^{-1}]$ as

$$b(\bar{\chi}) = {\rm erf}\left(\frac{1}{\sqrt{2}}\frac{\bar{\chi}}{\alpha}\right) - {\rm erf}\left[\frac{1}{\sqrt{2}}\frac{(\bar{\chi}+\tau_{\rm r}^{-1})}{\alpha}\right] - {\rm erf}\left[\frac{1}{\sqrt{2}}\frac{(\bar{\chi}-\tau_{\rm r}^{-1})}{\alpha}\right]\,. \tag{30}$$

This function is very close to +1 or -1 except in the neighborhood of the origin (or the points $n\tau_{\rm r}^{-1}$) where it is linearly dependent on $\bar{\chi}$:

$$b(\bar{\chi}) = \sqrt{\frac{2}{\pi}}\frac{\bar{\chi}}{\alpha}\,, \qquad \bar{\chi} < \alpha\,. \tag{31}$$

In Fig. 2(a) we show the bias function for several values of α.

When $\beta \gg 1$, we have that $\phi_0 \sim \lambda^{-1/4}H_{\rm i} \gg v$, and the initial values of $\bar{\chi}$ will be distributed with equal probability in the interval $[-\tau_{\rm r}^{-1}, \tau_{\rm r}^{-1}]$. Thus, in this case there is no dependence on β, and the results are relatively insensitive to the initial probability distribution for $\bar{\chi}$, even if stochastic equilibrium is not achieved during inflation. Therefore, the bias probability function $P(b)$ can be simply calculated using $P_b(b) = \frac{\tau_{\rm r}}{2}\frac{{\rm d}\bar{\chi}}{{\rm d}\,b}$ with $\bar{\chi}$ in the interval $[0, \tau_{\rm r}^{-1}/2]$, yielding

$$\begin{aligned} P_b &= \frac{1}{\pi}\left[\left(\frac{4}{\pi}\right)^2\exp\left(-\pi\tau_{\rm r}\alpha\right)^2 - b^2\right]^{-1/2} \quad \text{for} \quad \alpha \gtrsim 1\,, \quad |b| < \frac{4}{\pi}\exp\left(-\frac{\pi\tau_{\rm r}}{\sqrt{2}}\alpha\right)^2\,, \\ &= \sqrt{\frac{\pi}{8}}\tau_{\rm r}\alpha \quad \text{for} \quad \alpha \ll 1\,, \quad |b| \lesssim 1\,, \end{aligned} \tag{32}$$

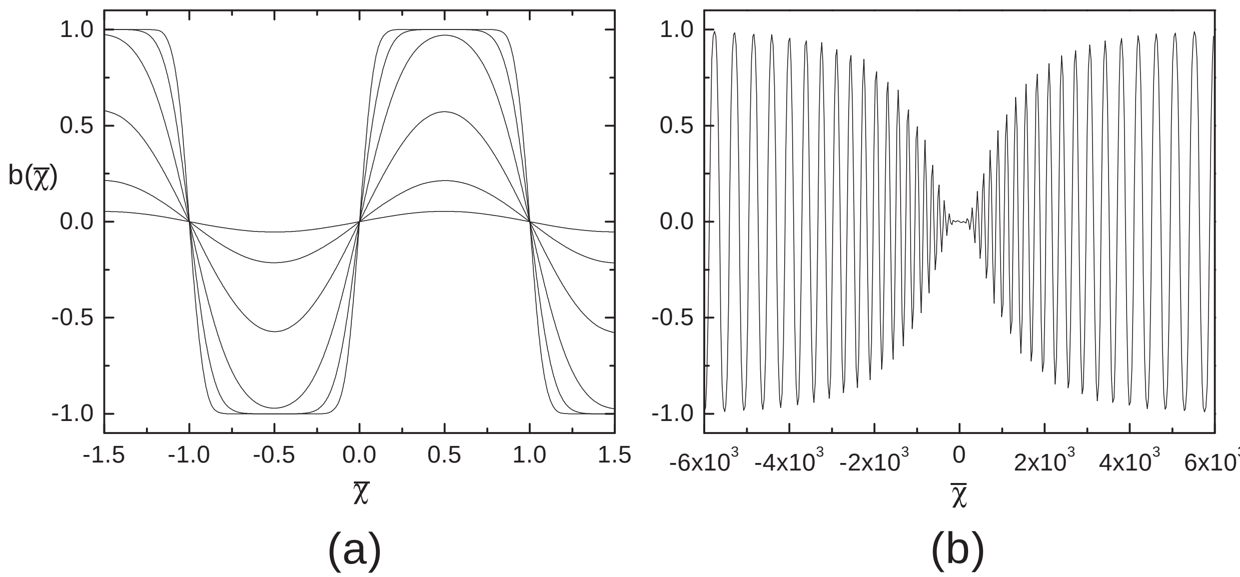

FIG. 2. The bias function for different values of α in (a) the radiation-dominated and (b) the matter-dominated case. In the left panel, the curves correspond, from top to bottom, to the values $\alpha = 0.05, 0.1, 0.2, 0.4, 0.6, 0.8,$ while on the right, the bias is shown for $\alpha = 100$.

where we have shown the maximum value the bias can reach (see Fig.3(a)). The cumulative probability for the bias is $P(|b| < x) = 2\tau_{\rm r}\bar{\chi}(x)$, where the function $\bar{\chi}(x)$ is just the inverse of the bias function in the interval $[0, \tau_{\rm r}^{-1}/2]$. This gives the approximate answer

$$\begin{aligned} P(|b| < x) &= \frac{2}{\pi}\arcsin\left[\frac{\pi x}{4}\exp\left(\frac{\pi\tau_{\rm r}}{\sqrt{2}}\alpha\right)^2\right] \quad \text{for} \quad x < \frac{4}{\pi}\exp\left(-\frac{\pi\tau_{\rm r}}{\sqrt{2}}\alpha\right)^2, \quad \alpha \gtrsim 1\,, \\ &= \sqrt{2\pi}\tau_{\rm r}\alpha\, x \quad \text{for} \quad x \lesssim 1, \quad \alpha \ll 1\,. \end{aligned} \tag{33}$$

This is plotted in Fig. 3(a) for various values of α.

2. Matter dominated universe

For the matter-dominated universe we have $n = \tau_{\rm m}\sqrt{\chi_0}$, and the bias will not be a periodic function of $\bar{\chi}$. Consequently the bias probability function is more sensitive to the initial probability distribution of $\bar{\chi}$, in contrast to the radiation-dominated case. Taking the stochastic distribution as the initial probability distribution for $\bar{\chi}$ the problem has two parameters rather than just one as in the radiation-dominated case.

Of course, the bias function $b(\bar{\chi})$ depends only on α. If $\alpha \ll 1$ then the bias will be concentrated around ± 1 since the width of the distribution will not allow different final vacuum states to be reached. The function will be ± 1 except near the transition points $\bar{\chi}_n = \tau_{\rm m}^{-2} n^2$, where it jumps as $\pm\mathrm{erf}(\frac{\bar{\chi}-\bar{\chi}_n}{\sqrt{2}\alpha})$. In fact this will also be the case for values of $\bar{\chi}$ greater than α^2 (with $\alpha \gtrsim 1$), because at this point the spacing of χ_0 which results in different final states is $\gtrsim \alpha$. In the opposite case, $\bar{\chi} \lesssim \alpha^2$, significant cancellation take place and the bias will be reduced. An excellent approximation in this regime (if $\alpha \gtrsim 1$) is given by the formula (29), if we allow for variations in the period and the amplitude of the sine function according to the effective variation of α with $\bar{\chi}$ (taking into account the increasing distance between the $\bar{\chi}_n$ with n in the matter dominated case), i.e.

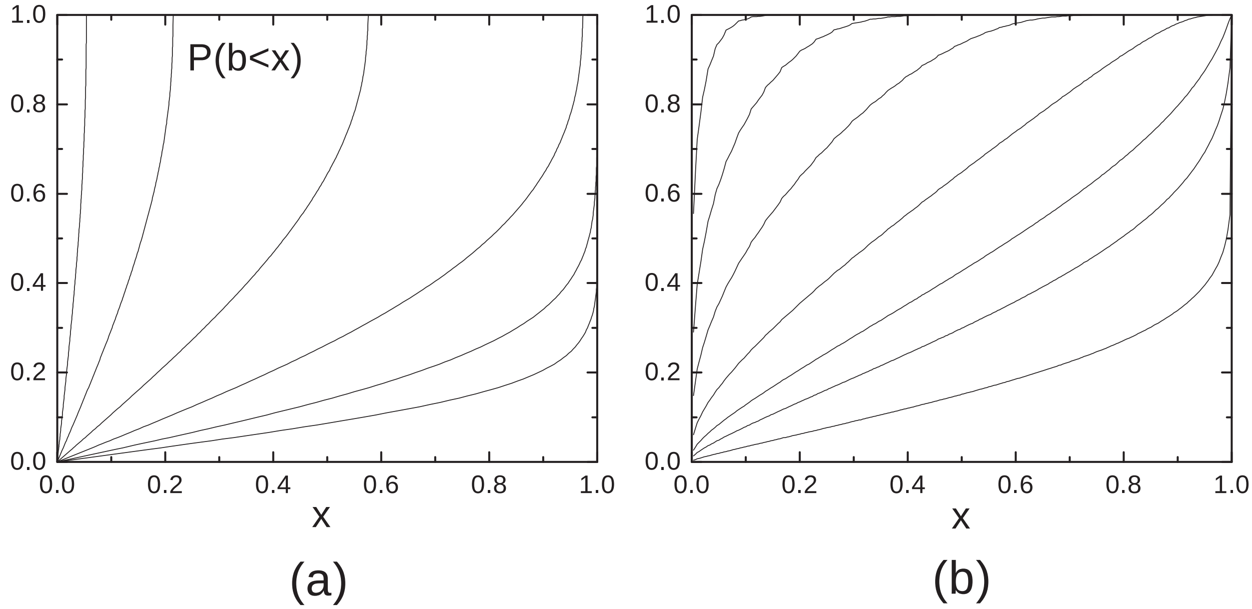

FIG. 3. The cumulative probability in (a) the radiation-dominated and (b) the matter-dominated case. In the left panel, the curves correspond, from bottom to top, to the values $\alpha = 0.05, 0.1, 0.2, 0.4, 0.6, 0.8$ (as in Fig. 2(a)), while in the right panel the curves correspond, from bottom to top, to the values $\alpha/\sqrt{\beta} = 0.2, 0.45, 0.7, 1.0, 1.6, 2.2, 3.2$.

$$b(\bar{\chi}) = \frac{4}{\pi}\mathrm{sgn}(\bar{\chi}) \exp\left[-\frac{1}{|\bar{\chi}|}\left(\frac{\pi\tau_{\mathrm{m}}\alpha}{2\sqrt{2}}\right)^2\right] \sin\left(\pi\tau_{\mathrm{m}}\sqrt{|\bar{\chi}|}\right), \quad \text{for} \quad \bar{\chi} \lesssim \alpha^2 . \tag{34}$$

In Fig. 2(b) we show an example of the bias function for the matter-dominated case.

For the case of interest, i.e. $\beta \gg 1$, the mass term can be neglected in Eq.(10) and the initial probability for $\bar{\chi}$ writes

$$P_{\bar{\chi}} = \frac{\sqrt{\pi}}{2^{3/4}3^{1/4}\Gamma(5/4)\beta} \exp\left(-\frac{2\pi^2}{3}\beta^{-4}\bar{\chi}^4\right) . \tag{35}$$

This function is practically constant for $\bar{\chi} < \beta/2$ and then falls until $\bar{\chi} = \beta$ where it is nearly zero. Thus for $\beta \gg \alpha^2$ the bias is ± 1 with high probability because $\bar{\chi} \sim \beta$. In this case we recover the linear behavior (33) of the cumulative probability for the bias in the radiation-dominated case, but now the spacing τ_{r}^{-1} depends on β:

$$P(|b| < x) = \sqrt{2\pi}\frac{\alpha}{0.71\sqrt{\beta}}\, x , \quad \text{for} \quad x \lesssim 1, \quad \alpha \ll \sqrt{\beta} , \tag{36}$$

where the precise coefficient in the identification $\tau_{\mathrm{r}}^{-1} \sim \sqrt{\beta}$ is $\frac{2^{5/8}\Gamma(9/8)}{\sqrt{5}\Gamma(5/4)} \simeq 0.71$, as can be calculated from the average value of the inverse spacing.

For $\alpha < \beta \lesssim \alpha^2$ the bias has higher probability between -1 and 1. In this case the probability $P(|b| < x)$ is dominated by the sector in the bias function (34) where $x > \frac{4}{\pi} \exp\left[-\frac{1}{|\bar{\chi}|}\left(\frac{\pi\tau_{\mathrm{m}}\alpha}{2\sqrt{2}}\right)^2\right]$ and thus $|\bar{\chi}| < \bar{\chi}_x = -(\frac{\pi\tau_{\mathrm{m}}\alpha}{2\sqrt{2}})^2/\log(\frac{\pi}{4}x)$. Then

$$P(|b| < x) = 2\int_0^{\bar{\chi}_x} P_{\bar{\chi}}\, \mathrm{d}\bar{\chi} = 1 - \frac{\Gamma(\frac{1}{4}, \beta^{-4}\bar{\chi}_x^4)}{\Gamma(\frac{1}{4})} , \quad \text{for} \quad \sqrt{\beta} \lesssim \alpha , \tag{37}$$

where the function $\Gamma(x,y) = \int_y^\infty t^{x-1}\mathrm{e}^{-t}\mathrm{d}\,t$ is the incomplete gamma function. The behavior of the cumulative probability function changes from linear to the form (37) as β/α^2 becomes smaller and in both limits it depends only on this combination of the parameters as shown in Fig. 3(b).

Let us recapitulate the main features. In the radiation-dominated case there is no dependence on β, and the probability of a small bias b is just $\sim b\alpha$ for $\alpha \sim H_\mathrm{i}/v \ll 1$, while for $\alpha \gtrsim 1$ it increases exponentially towards unity as $\sim b\mathrm{e}^{\alpha^2}$. For the matter-dominated universe, the parameter that regulates the bias probability and plays the same role as α in the radiation-dominated case is $\alpha/\sqrt{\beta} \sim \sqrt{\lambda^{1/4}H_\mathrm{i}/v}$. If $\beta \equiv \lambda^{-1/4}H_\mathrm{i}/v \gg \alpha^2$ the probability of a small bias is also small, $\sim b\alpha/\sqrt{\beta}$, but if $\beta \lesssim \alpha^2$ it increases exponentially fast, being near unity when $b\mathrm{e}^{\alpha^2/\beta} \sim 1$.

In a general situation where the number of half-oscillations n is some function $n(\phi_0)$, the change in the initial field necessary to change the number of half-oscillations by unity is

$$\Delta\phi_0 = \left[\frac{\mathrm{d}n(\phi_0)}{\mathrm{d}\phi_0}\right]^{-1}. \tag{38}$$

This needs to be compared with the width of the initial distribution $\sigma \simeq H_\mathrm{i}$ in order to estimate the bias. If $H_\mathrm{i}\mathrm{d}n(\phi_0)/\mathrm{d}\phi_0 \gg 1$ the bias will be exponentially damped and the domain wall network will be stable. On the contrary, if $H_i\mathrm{d}n(\phi_0)/\mathrm{d}\phi_0 \ll 1$ the bias will be of order unity and the subsequent evolution will make the wall network collapse exponentially fast [16–18].

V. HISTORY OF THE FIELD ENERGY DENSITY AND WALL FORMATION

The history of the energy density of the coherent field component for the Z_2-symmetry breaking potential (1) is approximately as follows. During inflation the energy density in the ϕ field is of $\mathcal{O}(H_\mathrm{i}^4)$, and the value of the field is $\phi_0 \sim 0.3\lambda^{-1/4}H_\mathrm{i}$ in stochastic equilibrium. (We have assumed that the quartic term in the potential dominates so that $H_\mathrm{i} > h$, where h is the height of the potential barrier; this is a necessary condition for inflation to produce domain walls, since otherwise the field will be settled in one of the two minima both during and after inflation.) If the symmetry is restored by thermal effects following reheating after inflation then domain walls will form again by the Kibble mechanism [2]. However if the field is sufficiently weakly coupled this will not happen [28] and $H_\mathrm{i} < h$ will then be a sufficient condition for inflation to solve the wall problem.[2]

After inflation, the field perturbations do not evolve significantly until H becomes less than $\omega \sim \lambda^{1/2}\phi_0 \sim 0.3\lambda^{1/4}H_\mathrm{i}$. Subsequently the field starts oscillating. At this point its energy density is still $\sim 0.01H_\mathrm{i}^4$ while the energy density of the universe is $\sim 3M_\mathrm{P}^2H^2 \sim 0.3\lambda^{1/2}H_\mathrm{i}^2M_\mathrm{P}^2$. Thus for the field ϕ not to dominate the energy density we require $\lambda^{1/2}M_\mathrm{P}^2 > 0.03H_\mathrm{i}^2$, i.e. ϕ_0 should not significantly exceed M_P, which seems natural. If $\lambda < 10^{-3}(H_\mathrm{i}/M_\mathrm{P})^4$ the stationary stochastic distribution for the initial conditions will not apply.

The energy density of the oscillating field redshifts as radiation hence its relative energy, compared with the total, either decreases or remains constant depending on whether the universe is matter-dominated or radiation-dominated. Therefore the energy density of the field can *never*

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[2]In principle symmetry restoration can also occur through non-thermal effects during "preheating" leading to the formation of topological defects [29] — however according to the results of Ref. [30] this will not happen for the model under consideration here.

dominate since it did not do so initially when the field was began to oscillate at the end of inflation. However after the domain walls form, the relative energy density starts increasing again (at least until the wall network decays due to the bias). The field is released when $t = t_* \sim (\lambda^{1/4} H_{\mathrm{i}})^{-1}$ and subsequently its energy density decays as $\rho_\phi = (R_*/R)^4 H_{\mathrm{i}}^4$. The walls will form when ρ_ϕ decreases below h^4 i.e. at a time t_{w} determined by

$$\frac{R(t_{\mathrm{w}})}{R(t_*)} \sim \frac{H_{\mathrm{i}}}{h} \,. \tag{39}$$

Using the results of the preceding sections we show in Fig. 4 the outcome for wall formation and survival in the parameter space of the scalar field model (1), for two specific set of inflationary parameters — (a) $H_{\mathrm{i}} = 2 \times 10^4$ GeV corresponding to an inflationary scale of $\sim 10^{11}$ GeV, $T_{\mathrm{reh}} = 10^3$ GeV, and (b) $H_{\mathrm{i}} = 10^{15}$ GeV corresponding to an inflationary scale of $\sim 10^{16}$ GeV, $T_{\mathrm{reh}} = 10^9$ GeV. Reheating is assumed to occur while the inflaton oscillates in a quadratic potential, so the scale-factor evolves as for a matter-dominated universe and reheating ends at $t_{\mathrm{reh}} \simeq M_{\mathrm{P}}/T_{\mathrm{reh}}^2$.

VI. DISCUSSION

We have developed the tools for computing the bias in domain wall formation in a specific Higgs-like model with a Z_2-symmetry. As a general rule we find that the quantity which controls the bias is $H_{\mathrm{i}}\,(\mathrm{d}\,n(\phi_0)/\mathrm{d}\phi_0)$, where the function $n(\phi_0)$ gives the total number of half-oscillations performed by the field, starting with the value ϕ_0 at the end of inflation, until it settles in one of its symmetry-breaking minima. This has to be evaluated taking into account both the form of the scalar potential and the expansion history of the universe. An exponentially small bias corresponds to $H_{\mathrm{i}}\,(\mathrm{d}\,n(\phi_0)/\mathrm{d}\phi_0) \gtrsim 1$ while the bias increases approximately linearly for $H_{\mathrm{i}}\,(\mathrm{d}\,n(\phi_0)/\mathrm{d}\phi_0) < 1$. To obtain the probability distribution of the bias, one needs the initial probability distribution for ϕ_0. We have used the stochastic description [19] of the field fluctuations during inflation to compute this.

The results of our detailed study for the Higgs-like potential are shown in Fig. 4 adopting both a high (10^{16} GeV) and an intermediate (10^{11} GeV) energy scale for inflation. We see that stable domain wall formation does not occur in most of the parameter space. For intermediate-scale inflation, the region where a problematic stable domain wall network forms is much smaller than for GUT-scale inflation and the domain wall problem is practically eliminated in this case.

The results of this paper can be applied to other models as well. For a harmonic potential as in the case of an axion field we recover the results of Ref. [7]. For a periodic potential the distribution of initial values of $\bar{\phi}$ is not relevant and we have a situation similar to the case of a Higgs-like potential in a radiation-dominated universe which was studied in Section IV. The parameter that controls the bias in this case is H_{i}/f where f is the period of the potential (i.e. the scale of Peccei-Quinn symmetry breaking for the axion field).

In Ref. [12] the authors consider domain wall formation in a model with a Higgs-like potential for a dilatonic-type scalar field with a *very* small coupling constant, $\lambda \sim 10^{-88}$, and a vacuum expectation value $v < 10^{11}$ GeV. As the authors acknowledge, the issue of domain wall formation is somewhat subtle in this extremely weakly coupled theory. For the model considered in the present work, such couplings imply wall formation around the present epoch or later (see Fig.4), so they would not in fact be astrophysically relevant. Furthermore the walls are formed in the high

bias region unless $H_{\rm i} \gtrsim 10^{13}$ GeV. However unlike the model considered in this work, the dilatonic field of Ref. [12] also couples universally with matter through the term

$$\mathcal{L}_{\rm int} = \exp\left(\frac{\phi}{M^*}\right)\theta^\mu{}_\mu, \tag{40}$$

where θ^μ_μ is the trace of the energy-momentum tensor and $M^{*2} \gtrsim (10^3 - 10^4)M_{\rm P}^2$. During inflation the trace θ^μ_μ is non-zero, driving the field quickly to large negative values $\phi \lesssim -70M^*$ (where the exponential factor in Eq.(40) makes the size of this interaction term comparable to the Higgs-like potential). Thus the effective mass is much smaller than $H_{\rm i}$ and the generation of fluctuations of size $H_{\rm i}$ is unavoidable. After inflation the term (40) decreases rapidly, so the field must relax in the quartic potential alone, starting at scales higher than the Planck mass. As we have previously discussed the absence of a force term strong enough to drive the field to the origin will make the potential energy of the field dominate the energy density of the universe. In Ref. [12] the authors also introduce a term that couples the field coherently to the thermal bath during the radiation-dominated era,

$$V_{\rm therm} = \frac{\kappa}{2}H^2\phi^2 \sim \frac{T^4}{M_{\rm P}^2}\phi^2\,, \tag{41}$$

where κ is a numerical constant. The intention in doing so is to drive the field to the origin, restoring the symmetry, so domain walls would be formed when H falls below the (vacuum) mass of the field. However, if ϕ is initially greater than the Planck mass this term will exceed the energy density of the universe so the treatment is not consistent. This problem does not occur if $\kappa \ll 1$ but in that case the mass of the field is *always* much smaller than H so the thermal term cannot affect the field.

Apart from this problem with the initial conditions, it is interesting to examine the effect of the coupling (41) with the thermal bath on the damping of the oscillations. The equation of motion for the field in the radiation-dominated universe with such a potential term is exactly solvable, with the general solution

$$\phi(t) = c_1 t^{-(1+\sqrt{1-4\kappa})/4} + c_2 t^{-(1-\sqrt{1-4\kappa})/4}. \tag{42}$$

For $\kappa \ll 1/4$ the friction dominates and the field decreases very slowly as $t^{-\kappa/2}$ (dominant term in Eq.(42)), while for $\kappa > 1/4$ the behavior is oscillatory and the amplitude decreases as $t^{-1/4} \sim R^{-1/2}$. Thus in either case the field amplitude decreases more slowly than for the quartic potential we have considered, where $\phi \sim R^{-1}$, or even for the quadratic potential, where $\phi \sim R^{-3/2}$. This makes it even more unlikely that the walls can be formed before the present epoch. Thus this interesting attempt to do away with dark matter in galaxies by modifying the gravitational force law cannot work.

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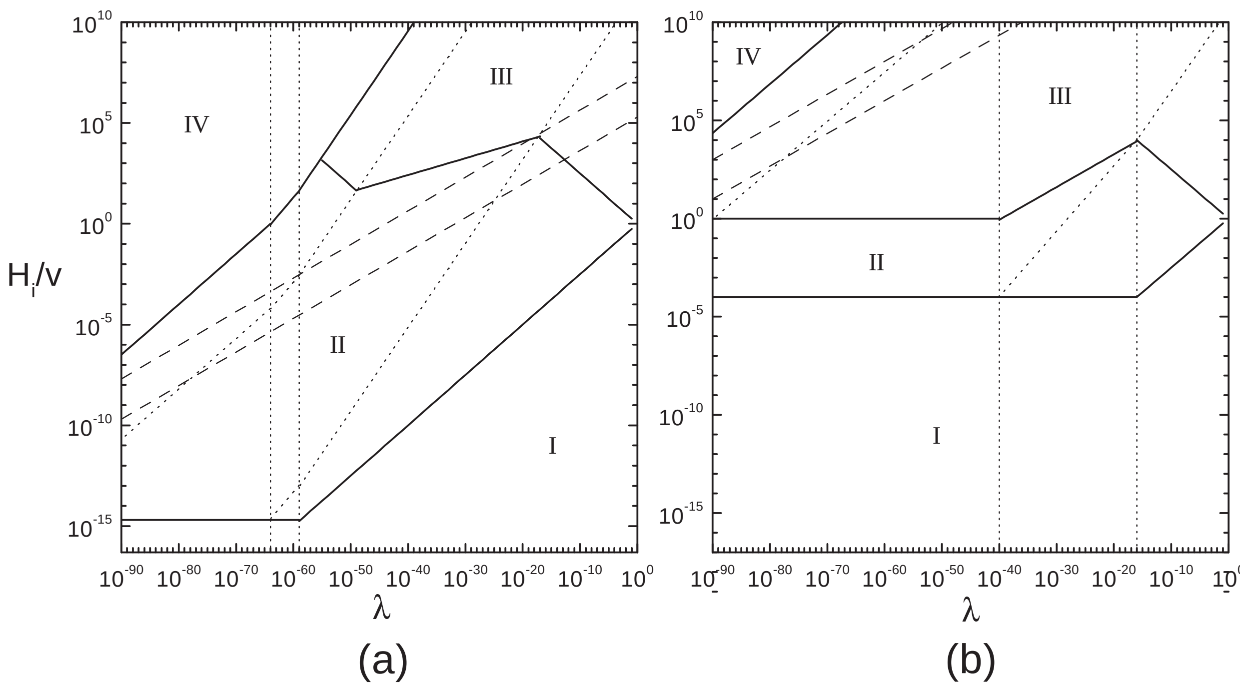

FIG. 4. The schematic situation concerning wall formation in the parameter space of the model (1) discussed in this work. In region I the inflationary perturbations are too small to jump the potential barrier and the symmetry remains always broken. In regions II and III walls form with, respectively, high and low bias. In region IV the walls have not yet formed by the present time and the field is still oscillating. The walls form in the matter-dominated era above the upper dotted line, during the reheating epoch below the lower dotted line, and during the radiation-dominated era in the region in between. The left vertical dotted line separates regions where the oscillations begin at the reheating epoch (right side) or in the radiation-dominated era (left side). The right vertical dotted line separates regions where stochastic equilibrium applies (right side) or where different initial conditions apply (we have conservatively assumed $\phi_0 \sim M_\mathrm{P}$ for the initial condition in this region). The dashed lines indicate the domain wall tension σ corresponding to wall domination at present ($\sigma^{1/3} \simeq 100$ MeV, lower line), and excessive CMB anisotropy ($\sigma^{1/3} \simeq 1$ MeV, upper line). Above these lines the tension is low enough to avoid conflict with observations. In the neighborhood of the line separating regions II and III, the bias takes values between zero and unity. In region III it approaches zero exponentially fast so the wall system survives. In region II the bias is large and the wall network collapses so there is no cosmological domain wall problem. The inflationary parameters are (a) $H_\mathrm{i} = 2 \times 10^4$ GeV, $T_\mathrm{reh} = 10^3$ GeV, and (b) $H_\mathrm{i} = 10^{15}$ GeV, $T_\mathrm{reh} = 10^9$ GeV.